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RESEARCH ARTICLE

Right angled triangles having equal area

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Abstract

In math literature there is mention of the above topic. Even Euler and others have given parametric solutions (see Dickson, 1919). There are some parametric solutions on the internet (Stack Exchange, n.d.-a, n.d.-b, n.d.-c, n.d.-d, n.d.-e). In this paper the authors have provided alternate methods to find parametric solutions. While some authors have given solutions of degree three and higher, the present paper provides solutions of degree five and degree two.

Since two right angled triangles can be represented by:

$$(p,q,r) = (2ab,\ a^2-b^2,\ a^2+b^2) \text{and} (s,t,u) = (2cd,\ c^2-d^2,\ c^2+d^2)$$

And, if their areas are to be equal, then we have to fulfil the condition:

$$ab(a^2-b^2) = cd(c^2-d^2) \qquad (1)$$

Equation (1) is a quartic equation of degree four.

Method (1)

We have:

$$AB(A^2-B^2) = CD(C^2-D^2)$$

We take:

$$A = pt + a, B = qt + b, C = pt + c, D = qt + d$$

To eliminate the constant term, we determine (p,q) and we get an equation in t.

$$p = n^3 + 3n^2 - 3n - 1$$
$$q = n^3 - 3n^2 - 3n + 1$$

$$t = \frac{[-3(n^2+1)]}{[16n(n^2-1)]}$$

This allows us to derive the parametric solution
of degree five.

$$(a,b,c,d) = \begin{bmatrix} su(n^2+4n-3), tv(3n^2+4n-1), sv(3n^2-4n-1), \\ tu(n^2-4n-3) \end{bmatrix}$$

$$where: (s,t,u,v) = [(n-1),(n+1),(3n^2-1),(n^2-3)]$$

This allows us to derive the parametric solution of degree five:

$$(a,b,c,d) = ((n-1),\ (n+1),\ (3n^2-1),\ (n^2-3))$$

Thus we have derived a parametric solution of degree five.

For $n = 3$ we get:

$$(a,b,c,d) = (76,\ 39,\ 52,\ 7)$$

Method (2)

Another parametric form is given by:

$$\begin{aligned} a &= p(p-q) \\ b &= -(p+q)(p+2q) \\ c &= (p+q)(2p+q) \\ d &= q(p-q) \end{aligned}$$

Above is a parametric form of degree two.

For $(p,q) = (7,5)$ we get:

$$(a,b,c,d) = (7,\ 102,\ 114,\ 5)$$

Method (3)

We have $ab(a^2-b^2) = cd(c^2-d^2)$. Taking $a = c$ and after some algebra we get:

$$a^2 = b^2 + bd + d^2 \qquad (2)$$

Equation (2) has the parametric solution:

$$a = m^2 + mn + n^2, b = m^2 - n^2, d = n^2 + 2mn$$

Since $c = a$, we obtain:

$$c = m^2 + mn + n^2$$

Hence,

$$a = m^2 + mn + n^2, b = m^2 - n^2, c = m^2 + mn + n^2, d = n^2 + 2mn$$

The above is a second-degree parametric solution.

For $(m, n) = (3,2)$ we get:

$$(a, b, c, d) = (19,\ 5,\ 19,\ 16)$$

Method (4)

We have $ab(a^2 - b^2) = cd(c^2 - d^2)$ – equation (1). We take:

$$a = \frac{5u^2+k}{u}, b = \frac{5u^2-k}{u}, c = \frac{3u^2+k}{v}, d = \frac{3u^2-k}{v} \tag{3}$$

After substituting into equation (3) and some algebra we get:

$k = uvw$

$$w^2(5u^2 - 3u^2) = (125u^2 - 27v^2) \tag{4}$$

Equation (4) has two solutions:

$$(k, v, w) = (2u^2,\ 2u,\ 1) \tag{5}$$

$$(k, v, w) = (7u^2,\ u,\ 7) \tag{6}$$

Thus for (5) we get $(a, b, c, d) = (7,\ 3,\ 7,\ 5)$ and for (6) we get:

$$(a, b, c, d) = (6,\ 1,\ 5,\ 2)$$

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